Algorithm of transformation between the local system Borowa Góra and the state system PL-2000 on the example of PG Silesia

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Abstract. The algorithm of the transformation task, obtained as a result of the realization of research concerning the accuracy evaluation and determination of coordinate transformation parameters between the local and state systems was presented. The algorithm was used practically to realize the transformation task between the local Borowa Góra system and the state system in the area of PG Silesia. Optimal transformation results were obtained using both the second-degree complex polynomial and the general-polynomial transformation of the second degree. The control of the accuracy of transformation showed that the accuracy of transformation and determination of transformation parameters is satisfactory. The above procedure proved that the two-stage transformation, which is currently in use in the mines, can be replaced by a one-step transformation conducted in accordance with the algorithm presented in the article.

1. Introduction

The current legal regulations regarding the keeping of surveyor-geological documentation [1], require a mining entrepreneur who conduct business requiring a concession, with the exception of a license granted by the district head, possession of surveyor-geological documentation. The documentation should be prepared in compliance with the provisions of the geodetic and cartographic law as well as the requirements specified in the Polish Standards – Mining Maps [2]. In Article 4, item 4 of the Regulation, the possibility of creating documents included in the documentation in the local geodetic systems was allowed, if the entrepreneur or entity conducting activity not requiring a concession has the possibility of geodetic transformation of this system to the geodetic reference system being a part of the national spatial reference system [3]. Research conducted in 2017 at the Silesian University of Technology [4]), proved that 28 out of the 29 mines, analyzed as a part of research, ran documentation in the Sucha Góra local system, in its different varieties, or the Borowa Góra system. In one case, the documentation was conducted in a 1965 flat rectangular coordinate system.

In connection with the above, the realization of the transformation task between the local system and the state system becomes extremely important, in particular the accuracy of transformation, due to the fact that these tasks relate to mining areas where we can observe surface deformations. Analysis of currently applied solutions of the transformational task in the mentioned mines proved that a solution that can be considered a two-stage transformation is applied. In the first stage, a transformation takes place between the local system and the 1965 state system, binding until the end of 2009, and then transformation to the currently applicable system, i.e. PL-2000. Another solution consists in a four-stage transformation used in the transition between the 1965 or the local system and the PL-2000 system, as discussed in detail in R. Kadaj [5]. In the research conducted at the Silesian University of Technology, the focus was on developing a methodology for the realization of the transformation task, allowing for the conversion of coordinates directly from the local system to the state system PL-2000. A transformation algorithm has been developed for this purpose, taking into account conducting the accuracy analysis of both the accuracy of determination of transformation parameters and verification of transformation. The article presents the developed algorithm and its practical application in the area of PG Silesia.

2. Algorithm of transformational task

The algorithm was developed assuming that it should allow transformation to be conducted at the required accuracy level, hence it also includes an assessment of the accuracy of determination of transformation parameters as well as the transformation of coordinates. The accuracy assessment was based on the provisions contained in the G-2 Manual – Detailed horizontal and altitudinal geodetic network and the conversion of the coordinates between systems [6], and technical guidelines G-1.10 [5]. It was assumed that the admissible transformation error should not exceed 0.05 m and it is most preferable to apply a solution using polynomial transformation, conformal and non-conformal of higher degrees. According to the provisions contained in the above documents, the adjustment points should be evenly distributed throughout the area subject (potentially) to transformation. Most preferably, they should be arranged so that the extreme points (edge points) form a convex figure covering the transformed area.

The developed transformational task algorithm includes:

- Determination of the optimal degree and a type of transformation based on the generated adjustment points, arranged in a regular grid, e.g. 250 x 250 m grid, located in the area covered by the transformation task, which ensures an error-free fit between the coordinate systems. The number of points should be selected empirically, so as to ensure a sufficient over-numerical of the system of equations.
- Performing an accuracy analysis, including an analysis of deviations of transformation on generated points. The most significant value should be the transformation error value, which should not exceed 0.05 m. We choose the degree and type of polynomial from the condition that we accept as the final the lowest degree of a polynomial, i.e. one at which the increase of a degree by one does not cause a significant decrease in the error of transformation. The transformation error is calculated from the formula:

$$S = \sqrt{\frac{[Sx^2]+[Sy^2]}{n-\frac{u}{2}}} \tag{1}$$

where:
Sx, Sy – deviations of the coordinates x, y for joint points,
n – the number of joint points
u – the number of unknown transformation parameters.

In order to perform a full accuracy analysis, it is recommended to calculate the medium-squared deviation Sx, Sy (2):

$$Sx = \sqrt{\frac{[Sx^2]}{n}},\ Sy = \sqrt{\frac{[Sy^2]}{n}} \tag{2}$$

and maximum deviation $Smax$ = max, the value is calculated by the formula (3):

$$S_i = \sqrt{(S_{xi}^2 + S_{yi}^2)} \tag{3}$$

where:

i=1,2,3…n,

n – the number of a joint point.

- The selection of joint points for transformation and accuracy analysis including deviations at the joint points, the maximum deviation and the transformation error. The transformation error calculated with the formula (1) should not exceed 0.05 m. For full accuracy analysis, it is recommended to calculate the values Sx, Sy with the formula (2) and $Smax$ with the formula (3).
- The determination and control of transformation parameters by analyzing discrepancies between coordinates obtained from the mine (Xp, Yp) and obtained as a result of transformation *(Xt, Yt)*. The analysis should include at least the *X (*d*x*), *Y (*d*y)* coordinates, and the resultant difference d*xy* (4):

$$\mathrm{d}xy = \sqrt{\mathrm{d}x^2 + \mathrm{d}y^2} \quad (4)$$

where:

dx=Xp-Xt

dy=Yp-Yt

- The verification of transformation accuracy using coordinates of the points from the area under transformation based on analysis of the coordinates divergence points *(Xp, Yp)* obtained from the mine or as a result of field measurements with the coordinates of these points calculated as a result of transformation *(X*t, *Y*t*)* of these points. The verification should include at least the *X (*d*x*), *Y (*d*y)* coordinates difference, and the resultant difference d*xy* (4).

3. The realization of the transformation task in the area of PG Silesia

The Mining Company Silesia derives directly from the former KWK Silesia, which was previously the action of a larger company, which included KWK Brzeszcze, [7, 8]. The company is located in Czechowice-Dziedzice and, as it stands, is owned by the Czech company *Energetický a průmyslový holding a.s.* The takeover took place on December 9, 2010, and the mining was resumed in April 2012 [8, 9].

The Borowa Góra system is used as the main coordinate system for conducting geological – survey documentation. In order to realize the transformation task, the following supplies were used: a map of the surface area obtained from PG Silesia, a survey of the PG Silesia mining level orientation, an archival survey from the PG Silesia mining level, a list of coordinates of the borderline breakdown of OG PG Silesia in the Borowa Góra and PL- 2000 systems, a list of the coordinates of the structure points in the Borowa Góra and PL- 2000 systems. To perform the transformation, the geodetic network points, the coordinates of which were determined both in the Borowa Góra system and the PL-2000, zone 6, were used as well as the coordinates of the selected points in PG Silesia determined as the result of field measurement in the PL – 2000 system and the coordinates of these points in the local system obtained from the mines. These points were designated by the external company as the result of work conducted in order to orientate the 6^{th} level of PG Silesia.

3.1. Determination of the optimal type and degree of transformation polynomial coordinates between the systems of Borowa Góra and PL – 2000

Inside the PG Silesia mining area, a grid of approximately 1000 evenly spaced points was generated, which allowed to determine the type and degree of polynomial ensuring an error-free fit between the coordinate systems. The distance between the points in the grid is 250 m. The number of the above was chosen empirically. It provides a sufficient over-numerical of the system of equations. The grid was made in the Geolisp software (figure 1).

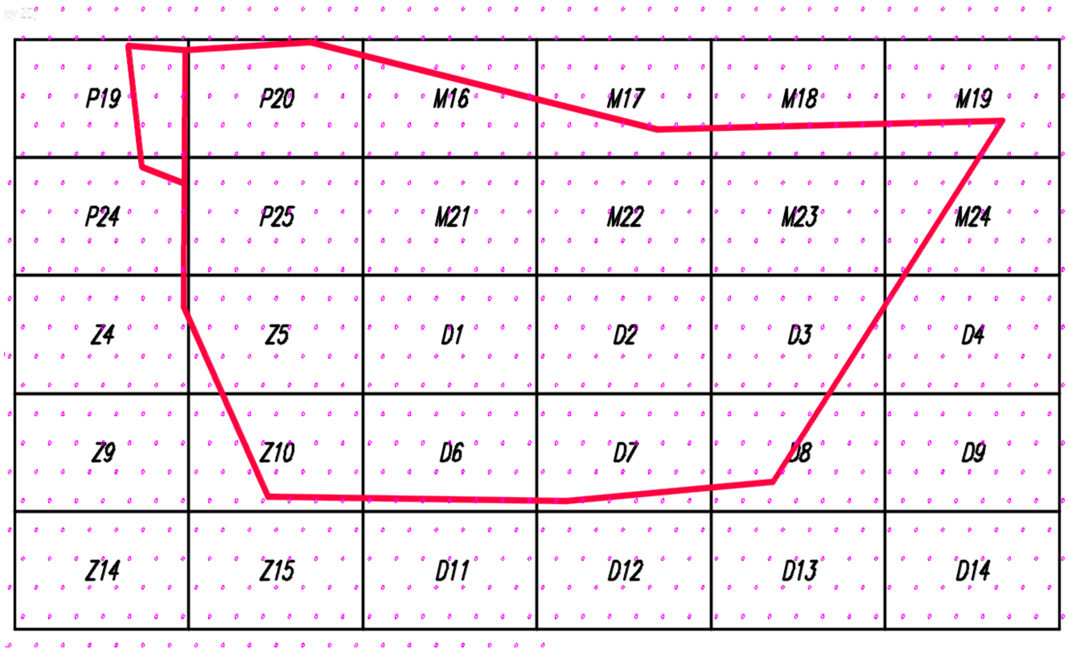

Figure 1. Coverage of the mining area of PG Silesia with a grid of test points (250m x 250m) used to pre-determinate the optimal type and grade of the transforming polynominal.

These points, with the coordinates in the Borowa Góra local system, were transformed to the PL – 2000 system using the Geolisp and Unitrans software, receiving a set of joint points in the local and national systems. For the generated set of joint points (adjustments) in the Geonet software, the general-polynomial and conformal (complex polynomial) transformation of a degree 1-3 was performed, resulting in the deviations presented in Table 1.

Table 1. The deviations of transformation of the flat coordinates between the Borowa Góra and PL 2000 zone 6 systems

A type and a degree of transformation	The values of deviation in [mm]			
	Sx	*Sy*	*Smax*	*S*
the conformal of the1st degree	9	7	29	12
the conformal of the 2nd degree	1	1	2	1
the conformal of the 3rd degree	1	1	2	1
general polynomial of 1st degree	9	7	29	12
general polynomial of 2nd degree	0	0	0	0
general polynomial of 3rd degree	0	0	0	0

The obtained results indicate that the optional one-stage transformation of the flat coordinates between the Borowa Góra and 2000 systems for the studied PG Silesia mining area is conformal or general-polynomial of the 2nd degree transformation.

3.2. Selection of joint points for transformation between the Borowa Góra and PL – 2000 systems

23 points of the basic structure from the level VI were obtained from the Silesian mine. For the reason that these points did not cover the entire mining area, the set of these points was supplemented by 45 points of the area and mining area (figure 2), which had the coordinates in both systems.

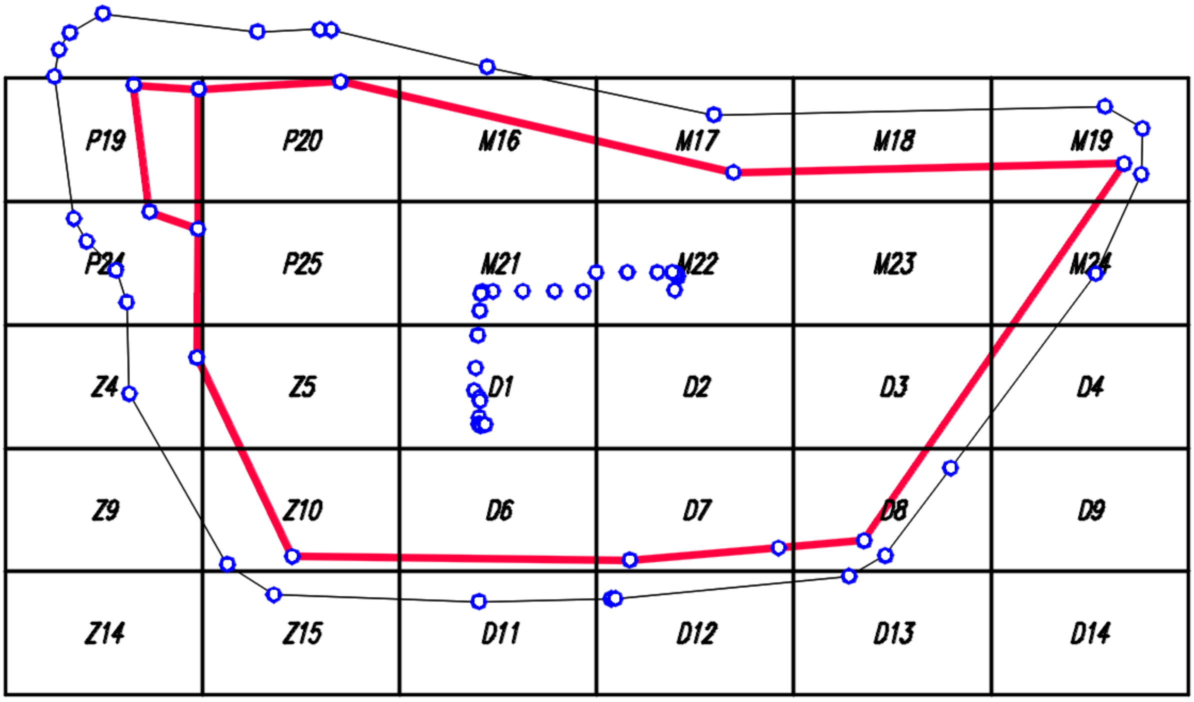

Figure 2. Selected joint points constituting the basis of the transformation algorithm between the Borowa Góra and PL-2000 systems.

For the such created set of the joint points, a conformal and the general-polynomial transformation of the 2nd degree were developed.

Table 2. Deviations of the flat coordinates transformation between the Borowa Góra and PL – 2000 systems

A type and a degree of transformation	The values of deviation in [mm]			
	Sx	*Sy*	*Smax*	*S*
the conformal of the 2^{nd} degree	6	8	17	10
general polynomial of 2^{nd} degree	5	5	14	7

The obtained results indicate that all investigated transformations meet the assumed requirement of 0.05m transformation error.

3.3. Determination of the transformation coefficients between Borowa Góra and PL – 2000 systems

For a one step transformation of the flat coordinates between the Borowa Góra and PL – 2000 systems, for the studied mining area of PG Silesia, the conformal transformation of the 2^{nd} degree was assumed. The determined transformation parameters from the Borowa Góra system, called the original to the PL – 2000 system that is called the secondary one, are presented below.

GENERAL PARAMETERS:

The centers of gravity of the sets of adaptation points:

The original system (1): 5534514.6432 6573764.7505 (xo, yo)

The secondary system (2): 5534620.4375 6573613.7378 (Xo, Yo)

The numeric scale = 1.66996377155748E-0004

THE COEFFICIENTS OF A COMPLEX POLYNOMIAL AND THE MEDIUM ERRORS

a[0]:= -7.93499902256937E-0005 a medium error = 2.20902922207219E-0005
b[0]:= -4.71021173736951E-0003 a medium error = 2.20902922207219E-0005
a[1]:= 5.98769841781197E+0003 a medium error = 3.55488424108825E-0005
b[1]:= -7.26142184519573E-0002 a medium error = 3.55488424108825E-0005
a[2]:= -7.01292895371615E-0004 a medium error = 5.50034514769903E-0005
b[2]:= -3.01726531388509E-0002 a medium error = 5.50034514769903E-0005

A TRANSFORMATION MODEL (a complex polynomial)

Z-Zo = c[0] + c[1]*z + c[2]*z*z + c[3]*z*z*z + ...
where:
Z= (X,Y) – the result coordinates,
Zo = (Xo,Yo) – the center of gravity In the secondary system,
z=[(x-xo)* scale, (y-yo)* scale] – complex argument,
(x,y)– the original coordinates,
(xo,yo)– the center of gravity In the original system,
c[0]=(a[0],b[0]), c1=(a[1],b[1]), ... ,c[n]=(a[n],b[n]) – polynomial coefficients.

3.4. The control of transformation parameters

To control the calculated transformation parameters, the structure points and the border points obtained from the mine were used. The figure 3 shows the ellipsis of the maximum discrepancy of the coordinates of the points in the second-degree conformal transformation between the systems of Borowa Góra and PL – 2000, in red, the points with a discrepancy greater than 40mm, in navy blue > 30mm, in blue > 20mm and in green the others are marked.

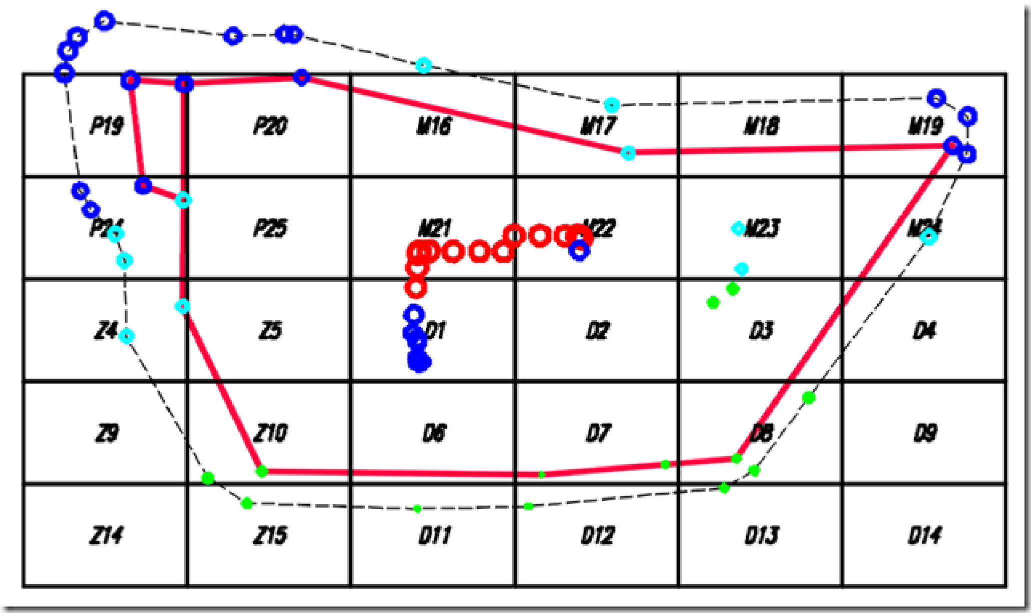

Figure 3. Ellipse of the discrepancies of the coordinates of the points in the second-degree conformal transformation between the Borowa Góra and PL – 2000 systems, (description in the text).

A list of the obtained discrepancies, including the absolute values of the minimum and maximum differences of *X*, Y (Min_{dx}, Min_{dy}, Max_{dx}, Max_{dy}) and the minimum and maximum of a resultant difference (Min_{dxy}, Max_{dxy}) are presented in Table 3.

Table 3. The coordinates discrepancies between the values obtained from conformal transformation and form the mine.

$\mathbf{Min_{dx}}$[mm]	$\mathbf{Max_{dx}}$[mm]	$\mathbf{Min_{dY}}$[mm]	$\mathbf{Max_{dY}}$[mm]	$\mathbf{Min_{dXY}}$[mm]	$\mathbf{Max_{dXY}}$[mm]
0	49	0	44	8	53

Table 3 shows that the maximum coordinate difference Max_{dXY} obtained as a result of transformation and acquired from the mine does not exceed 53 mm. In 95% of cases, the difference does not exceed 0.05 m. In three cases the differences exceeded 50 mm, which is 5% of the points. It can be concluded that the obtained discrepancies indicate that the accuracy of the transformation is satisfactory.

3.5. The verification of accuracy transformation for the selected points in the mining area

In order to verify the transformation, the field measurements in the area Of PG Silesia were also performed. The areas with compact buildings were selected for the measurement. The ground elements of the underground utilities were measured, i.e. the middle of the covers of the sanitary and sewer catch pits as well as the middle of the water pipes covers, which are a part of the situational details of the first precision group. The selection of the measuring points was made considering the following conditions:

- having the coordinates of the points in the Borowa Góra local system,
- an even distribution of the points in the measurements area.

The flat coordinates were determined in the spatial reference state system PL – 2000/6. The GNSS receiver was used to achieve this goal.

In order to determine the coordinates of the points, RTN (real Time Network) method was used, which consists in creating a virtual reference station near the GNSS receiver (after receiving the approximate position of the receiver). For such a station, corrections (the interpolation of corrections from several neighboring reference stations) are calculated, which are then sent as if they came from a single reference station. For the measurement, corrections made available by the private nationwide GNSS surface area network and Glonass – TPI NET pro were used. This network consists of 136 references stations, there are evenly distributed throughout Poland and guarantees full compliance of the system with the technical requirements of GUGiK.

The measurement was made in accordance with the technical recommendations of the Main Surveyor of the Country [10] in the scope of the measurement of the situational and height network points. The main additional conditions considered during the instrumental measurement included, among others:

- the position was determined based on a minimum of 6 GNSS satellites,
- the PDOP parameter should be less than 3.

The coordinates of 58 points were measured.

The distribution of coordinate differences (absolute values), obtained as a result of measurements with coordinates calculated as a result of transformation, is presented in figure 4.

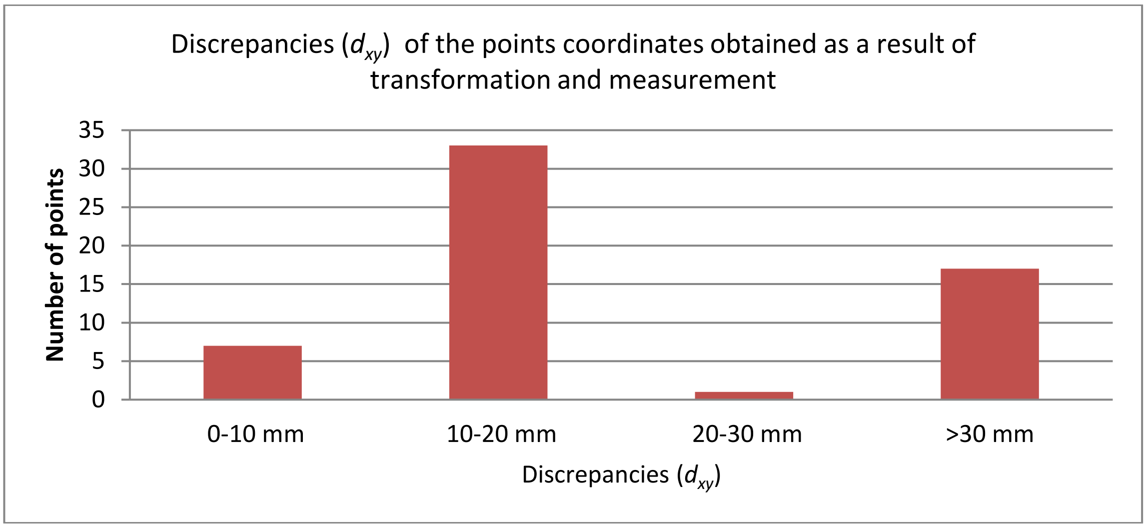

Figure 4. Distribution of discrepancies of the coordinates of measured and calculated points as a result of transformation

Table 4 presents the list of the obtained discrepancies, including the absolute values of the minimum and maximum differences of X, Y coordinates ($Min_{dx,}$ Min_{dy}, Max_{dx}, Max_{dy}), and the minimum and maximum of the resultant difference (Min_{dxy}, Max_{dxy}). The maximum coordinate difference Max_{dXY} does not exceed 40 mm. It can be concluded that the obtained discrepancies indicate that the accuracy of the transformation is satisfactory.

Table 4. Discrepancies in coordinates of the measured points obtained from transformation and field measurement.

$\mathbf{Min_{dx}[mm]}$	$\mathbf{Max_{dx}[mm]}$	$\mathbf{Min_{dY}[mm]}$	$\mathbf{Max_{dY}[mm]}$	$\mathbf{Min_{dXY}[mm]}$	$\mathbf{Max_{dXY}[mm]}$
2	27	7	29	9	39

4. Conclusion

Cartographic systems are realized through the geodetic networks and contain the stratification of various type of measurement and calculation errors. Therefore, these systems do not necessarily have to coincide with their theoretical formulas. Thus, despite the fact that the mathematical origin of the local system can be knowable (e.g. on the basis of archival information), only the adaptation points should be the basis for determining transformation formulas.

The transformation task related to the mining areas of mines is extremely important due to the fact that according to the provisions of the Regulation [6], mines are forced to perform transformation of the local coordinates to the state system. At present, this type of transformation is conducted essentially in two stages, through the transition from the local system to the 1965 system, and then from the 1965 system to the PL – 2000 system. The article presents the algorithm of a transformational task conducted in one stage, by converting coordinates from the local system directly to the PL – 2000 system. The algorithm includes a full assessment of accuracy and the development of transformation parameters. The above algorithm was used practically for the area of PG Silesia. It was found that:

- optimal transformation results for this area can be obtained by using both the complex polynomial of the second degree and the general – polynomial transformation of the second degree. In both areas deviations at the joint points did not exceed the assumed 0.05m error. Due to the fact that the transformation patterns between the systems should be based on the assumption of precision, the article presents the values of the determined parameters for the conformal transformation of the second degree;

- the control of transformation of accuracy proved that 95% of the points whose coordinates were controlled, discrepancies between the coordinates calculated as a result of the transformation and obtained from the mine did not exceed 0.05m;
- the verification of transformation accuracy based on the results of field measurements proved that the differences between the coordinates obtained as a result of measurements and calculated as a result of transformation did not exceed 0.05m, which confirms the implementation of the transformation at the assumed accuracy level;
- transformation parameters developed as a result of the work can be used in the transformation tasks related to the area of PG Silesia.

The transformation algorithm used in a smaller work can be used to determine the degree and a type of transformation and to determine the transformation coefficients for the remaining mines in the GZW region.

The accuracy of transformation of the border points is important in the transformation task. This issue should be the subject of further research.

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